MATHEMATICAL OPTIMIZATION: QUESTION LIST

Abstract. Some notes on organization and a list of questions helping you to prepare for the exam. At the exam, I want to see you writing formulas and hear you explain the concepts.

Organization

- 2h lectures + 2h Lab exercises per week.
- 80% lab attendance to pass the exercises.
- Exam: written or verbal exam; at most 1 retake.
- Bring your laptop to the lab.
- Office hours: Monday 2-3pm, Room 4.19
- felix.huber@ug.edu.pl to contact me by email.
  Write class (MathOpt) and surname in subject.

Material

(1) Lovász, lecture notes: Semidefinite programs and combinatorial optimization `https://sites.math.washington.edu/~thomas/teaching/m514_web/Lovasz_semidef.pdf`
(2) Blekherman et al, Semidefinite Optimization and convex algebraic geometry `https://www.mit.edu/~parrilo/sdocag/`
(3) Jason Li - Graph Matchings III: Weighted Matchings `https://www.cs.cmu.edu/~15850/notes/lec9.pdf`
(4) Heng Yang - Semidefinite Optimization and Relaxation `https://hankyang.seas.harvard.edu/Semidefinite/` (careful: different convention for primal and dual)
(5) Gupta, O'Donnel `https://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15859-f11/www/notes/lpsdp.pdf`
(6) Mironowicz, Semi-definite programming and quantum information `https://arxiv.org/abs/2306.16560`
(7) JuMP library for Optimization `https://jump.dev/`

1. Linear Algebra

1. Give definitions for the scalar product, outer product, trace inner product.
2. Define trace/rank of a square matrix.
3. What is a symmetric matrix, orthogonal matrix?
4. What is an eigenvector, eigenvalue; what are their properties for symmetric or positive semidefinite matrices?
5. What is the spectral decomposition? What is the Cholesky factorization?
6. Give five equivalent definitions for a matrix to be positive semidefinite.
7. For a matrix, what is the difference between positive and positive semidefinite?
8. What is the Löwner order?
9. What is a Gram matrix?
10. What is the positive cone?
11. Define convex cone, convex hull, dual cone, self-dual cone. Examples of self-dual cones?

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2. Linear Programming

12. Formulate the standard form of a linear program and its dual. What are its parameters, variables, objective functions?
13. What is a feasible point, a feasible set?
14. What is a polyhedron? A polytope? Give examples.
15. What is the non-negative orthant? The probability simplex? The unit simplex?
16. Explain Farkas Lemma and its use.
17. For an LP, what is weak duality, strong duality, duality gap?
18. What is complementary slackness?
19. Prove weak duality.
20. What is the duality theorem.
21. What is complementary slackness?
22. Assume the primal is feasible (infeasible, unbounded, optimal). What can we say about the dual (feasible/infeasible/optimal/unbounded)?
23. Explain s-t flow (max perfect matching, vertex cover); formulate an LP relaxation. What can we say about the relaxation?

3. Second order cone programming

24. Formulate the standard form of a second order-cone program.
25. Rank the computational complexity of solving LP/SOCP/SDP's.

4. Semidefinite Programming

26. Formulate the primal and dual standard formulations of semidefinite programs. What are the problem parameters?
27. For an SDP, what is weak duality, strong duality, duality gap? Prove weak duality.
28. What is a feasibility problem? What is an infeasibility certificate?
29. What is a spectrahedron? What is an elliptope?

5. Binary quadratic optimization

30. What is the Ising model? What is Max Cut? How are they related?
31. Formulate the binary quadratic optimization problem, and write the equivalent polynomial formulation.
32. Formulate SDP lower bounds on binary quadratic optimization problems (both primal and dual formulations). Explain the primal, derive its dual.
33. What is an approximation ratio?
34. How does the Goeman-Williamon algorithm work? What is its approximation ratio?

6. Lovasz ϑ number

35. What is an independent set?
36. What is the Hamming distance?
37. What is the repetition code, what are its parameters?
38. How is the Lovasz theta number defined as an SDP?
39. What is the strong graph product? What is $\vartheta(G \boxtimes H)$?
40. What is the Shannon capacity of a graph? How does the Lovasz θ bound it?
41. What is the capacity of the Pentagon?
42. Define independence number, chromatic number. What are their computational complexities?
43. What is Knuth's sandwich theorem? Why is it interesting/surprising?

7. Positive polynomials

44. What is a polynomial optimization problem?
45. What is a SOS proof?
46. I want to prove that the polynomial $p(x, y, z) = x^2 + y^4 + z^2 + 2xy^2 - 2y^2z - 2xz$ is non-negative on $\mathbb{R}$. How do I set up my SDP solver to show this?
47. Formulate the Putinar positivstellensatz.
48. Formulate the Lasserre hierarchy for polynomial optimization.
49. Give an example of a positive polynomial that is *not* sum of squares. Tell me more about it.
50. Give the geometric-arithmetic mean inequality.
51. What is the Newton polytope? How does one use it?

8. Advanced

52. What is the Grothendiek problem with rank constraint?