MATHEMATICAL OPTIMIZATION: LAB

Abstract. Tasks for the LAB

1. LAB 1

1. Install Julia + Jupyter Lab + JuMP solver framework + SCS solver and work through the linear programming Jump tutorial https://jump.dev/JuMP.jl/stable/tutorials/getting_started/getting_started_with_JuMP/

   Apart from Julia itself (and perhaps jupyter lab) you should be able to install most of this through the Julia package manager.

2. LAB 2

2. Check *strong duality* and *complementary slackness* for numerical solutions.
3. Dualize an infeasible LP and obtain infeasibility certificate

3. LAB 3

4. Impement LP relaxations for maximum perfect matching and vertex cover.
5. Implement the relaxations FSTAB, QSTAB for the stable set polytope.

4. LAB 4

6. Implement LP's for FSTAB and QSTAB.

5. LAB 5

7. Let $P = \mathrm{conv}\{(0,0),(2,0),(0,1),(2/3,2/3)\}$. Describe the Anti-blocker P^*.
8. Recall:

Theorem 1 (Chvatal for positive orthant)**.** *Given*

$$\min\{c^T x \,:\, Ax \leq b, x \in \mathbb{Z}^n_+\} \tag{1}$$

where $A \in Z^{m\times n}, b \in \mathbb{Z}^m, c \in \mathbb{R}$*; the inequality*

$$\lfloor \lambda^T A \rfloor x \leq \lfloor \lambda^T b \rfloor \tag{2}$$

is valid for P_I *for all* $\lambda \in \mathbb{R}^m_{\geq 0}$.

Task: Example from https://coral.ise.lehigh.edu/ ted/files/ie418/lectures/Lecture13.pdf
Make numerical iterative Gomory-Chvatal cuts for

$$\begin{aligned}
\max \quad & 2x_1 + 5x_2 && (3)\\
\text{s.t.} \quad & 4x_1 + x_2 \leq 28 \\
& x_1 + 4x_2 \leq 27 \\
& x_1 - x_2 \leq 1 \\
& x_1, x_2 \geq 0 && (4)
\end{aligned}$$

(Optimal basic feasible solution is: $x_1^* = 17/3, x_2^* = 16/3$ with $\alpha^* = 38$. Optimal IP solution is $\alpha^*_{IP} = 37.5$)

Can you find a numerical Gomory-Chvatal Closure P'?

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6. LAB 6

9. Compute eigenvalues, trace, scalar product, matrix product, Cholesky decomposition in Julia. Check their properties.

7. LAB 7

10. Study the Max Cut Note, and try to understand each line in the derivation (pen + paper, make examples). Implement the SDP.

8. LAB 8

11. Study the SOS Note, and try to understand each line in the derivation (pen + paper, make examples). Then find a decomposition of $5x^4 - 4x^3 - x^2 + 2x + 2$ as a sum of squares polynomial: first by hand and then by computer.

9. LAB 9

12. Compute $G \times H$ for two small graphs (e.g. line graph with $n = 3$ vertices) by hand. Implement an SDP for the Lovasz theta number. Compute $\vartheta(G^{\boxtimes 2})$ where G is the pentagon and $\boxtimes$ the strong graph product.

10. LAB 10

13. Write in standard form, formulate dual, and solve by hand

$$\begin{aligned} \min \quad & 2x_{11} + 2x_{12} \\ \text{s.t} \quad & x_{11} + x_{22} = 1 \\ & \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} \succeq 0 \end{aligned} \tag{5}$$

Solve the same problem but for max instead of min.

14. Solve or show infeasibility by unbounded dual:

$$\begin{aligned} \text{find} \quad & X \succeq 0 \\ \text{s.t} \quad & \langle A_1, X \rangle = 1 \\ & \langle A_2, X \rangle = 1 \\ & \langle A_3, X \rangle = 1 \end{aligned} \tag{6}$$

where

$$A_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

11. LAB 11

15. Implement the Goeman Williamson algorithm

12. LAB 12

16. Implement the Goeman Williamson algorithm (II)

13. Lab 13

17. Write a sum-of-squares solver: Input polynomial coefficients $(p_0, \dots, p_n)$, $n \in \mathbb{N}$.
    Find $A \succeq 0$ such that
$$p(x) = m^T A m$$
    where $m = (1, x, x^2, x^3, \dots)$.
18. Do polynomial optimization.
$$\begin{aligned} \min_{\lambda} \quad & \lambda \\ \text{s.t.} \quad & p - \lambda = \text{SOS} \end{aligned}$$